Динамика одиночной частицы в двухмерном стационарном потоке

Тукмаков Д.А., ИММ КазНЦ РАН

Казань 2024г.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial}{\partial x}\left(\rho u^2 + p\right) = 0 \quad (2)$$

$$\frac{\partial(e)}{\partial t} + \frac{\partial}{\partial y}\left(\left[e + p\right]u\right) = 0 \quad (3)$$

$$\frac{\partial c}{\partial t} = a\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial t} \quad (4)$$

$$\frac{\partial u_1}{\partial t} = \frac{F(u) - au_1}{m} \quad (5)$$

$$\frac{\partial \rho_E}{\partial t} + \frac{\partial\left(\rho_E u_E\right)}{\partial x} = 0 \quad (6)$$

$$\frac{\partial\left(\rho_E u_E\right)}{\partial t} + \frac{\partial}{\partial x}\left(\rho_E u_E^2 + p_E\right) = 0 \quad (7)$$

$$\frac{\partial\left(e_E\right)}{\partial t} + \frac{\partial}{\partial y}\left(\left[e_E + p_E\right]u_E\right) = 0 \quad (8)$$

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial (\rho_1 u_1)}{\partial x} = 0 \quad (9)$$

$$\frac{\partial \rho_2}{\partial t} + \frac{\partial (\rho_2 u_2)}{\partial x} = 0 \quad (10)$$

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial x}\left(\rho u^2 + p\right) = 0 \quad (11)$$

$$\frac{\partial (e)}{\partial t} + \frac{\partial}{\partial y}\left(\left[e + p\right] u\right) = 0 \quad (12)$$

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial(\rho_1 u_1)}{\partial x} = 0 \quad (13)$$

$$\frac{\partial(\rho_1 u_1)}{\partial t} + \frac{\partial}{\partial x}\left(\rho_1 u_1^2 + p\right) = -F + \alpha\frac{\partial p}{\partial x} \quad (14)$$

$$\frac{\partial(e_1)}{\partial t} + \frac{\partial}{\partial y}\left([e_1 + p]u_1\right) = -Q - |F|(u_1 - u_2) - \alpha\left(\frac{\partial(pu_1)}{\partial x}\right) \quad (15)$$

$$\frac{\partial \rho_2}{\partial t} + \frac{\partial(\rho_2 u_2)}{\partial x} = 0 \quad (16)$$

$$\frac{\partial(\rho_2 u_2)}{\partial t} + \frac{\partial}{\partial x}\left(\rho_2 u_2^2\right) = F - \alpha\frac{\partial p}{\partial x} \quad (17)$$

$$\frac{\partial(e_2)}{\partial t} + \frac{\partial}{\partial x}\left(e_2 u_2\right) = Q \quad (18)$$

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0 \qquad (19)$$

$$V_x=\frac{\partial\varphi}{\partial x}, \quad V_y=\frac{\partial\varphi}{\partial y} \qquad (19^*)$$

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}=0 \qquad (19^{**})$$

$$\frac{\partial V_{1i}}{\partial t}=\alpha\left(V_i-V_{1i}\right) \qquad (20)$$

$$\frac{\partial u_1}{\partial t}=\frac{a}{m}\left(\frac{\partial\varphi}{\partial x}-u_1\right) \qquad (21)$$

$$\frac{\partial v_1}{\partial t}=\frac{a}{m}\left(\frac{\partial\varphi}{\partial y}-v_1\right) \qquad (22)$$

$$\frac{\partial u_1}{\partial t}=\frac{36\mu}{\rho_1 d^2}\left(\frac{\partial\varphi}{\partial x}-u_1\right) \qquad (21^*)$$

$$\frac{\partial v_1}{\partial t}=\frac{36\mu}{\rho_1 d^2}\left(\frac{\partial\varphi}{\partial y}-v_1\right) \qquad (22^*)$$

$$u_1(t) = u_{10} \exp\left(-\frac{36\mu}{\rho_1 d^2} t\right) + \frac{\partial \varphi}{\partial x} \quad (23)$$

$$v_1(t) = v_{10} \exp\left(-\frac{36\mu}{\rho_1 d^2} t\right) + \frac{\partial \varphi}{\partial y} \quad (24)$$

Выводы.

В работе получена математическая модель динамики одиночной частицы в стационарном гидродинамическом поле несжимаемой жидкости.

Математическая модель демонстрирует, что по мере движения в стационарном потоке скоростные отличия движения частицы в сравнении с основным потоком уменьшаются.

СПАСИБО ЗА ВНИМАНИЕ!